HEAT TRANSFER IN THE NECK OF A VESSEL USED FOR STORING CRYOGENIC LIQUIDS

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The heat transfer characteristics are established on the basis of an analytic solution and experimental data. Relations are given for determining the axial heat flow along the gas-cooled neck.

We will consider a vessel containing the rising vapors of a cryogenic liquid. The top of the neck is at the temperature of the ambient medium T_0, and the bottom at the temperature of the cryogenic liquid T_L. The temperature of the gas at the inlet to the neck is also equal to T_L. The heat flow along the neck is determined by the temperature difference between its ends, the heat transfer to the outer surface through the layer of insulation from the ambient medium, and the heat exchange between the inner surface of the neck and the rising vapors.

The results of theoretical studies of the heat transfer in a neck are presented in [1–3]. In formulating the problem the authors neglect the heat transfer at the outer surface of the neck and assume that the heat exchange between the gas and the wall is perfect, i.e., $T_w = T_g$. In most cases the assumptions are too crude and the calculated values of the axial heat flows are in poor agreement with the experimental data [4].

We have solved this problem on the following assumptions:

1. The thermal conductivity of the neck material, the physical properties of the gas, and the thermal conductivity of the insulation at the outer surface of the neck do not depend on temperature.
2. The coefficient of heat transfer between the walls of the neck and the gas is assumed given and constant along the length.
3. There is no axial heat conduction through the gas.

The heat conduction equation for the walls of the neck and the energy equation for the gas flow now take the form

$$\lambda_w f \frac{d^2T_w}{dx^2} = \alpha\pi D_w (T_w - T_g) + \frac{2\pi\lambda_{in}}{\ln(D_{in}/D_w)} (T_w - T_g), \quad (1)$$

$$mc \frac{dT_g}{dx} = \alpha\pi D_w (T_w - T_g). \quad (2)$$

The boundary conditions can be written as follows:

$$x = 0, \quad T_w = T_0, \quad (3)$$

$$x = -L, \quad T_w = T_L, \quad (4)$$

$$x = -L, \quad T_g = T_L. \quad (5)$$

We introduce the dimensionless variables

$$X = \frac{x}{L}, \quad \Theta_w = \frac{T_w - T_g}{T_0 - T_L}, \quad \Theta_g = \frac{T_g - T_0}{T_0 - T_L} \quad (6)$$

and the dimensionless parameters

$$H = \frac{mc}{\lambda_w f} L, \quad (7)$$

$$K^2 = \frac{\alpha\pi D_w}{\lambda_w f} L = 4\mathrm{Nu} \left(\frac{L}{D_w}\right)^2 \frac{\lambda_g F}{\lambda_w f}, \quad (8)$$

$$\beta^2 = \frac{2\pi\lambda_{in} L^2}{\lambda_w f \ln(D_{in}/D_w)}. \quad (9)$$

Using (6)–(9), we reduce Eqs. (1) and (2) to dimensionless form:

$$\frac{d^2\Theta_w}{dX^2} = K^2(\Theta_w - \Theta_g) + \beta^2\Theta_w, \quad (10)$$

$$H\frac{d\Theta_w}{dX} = K^2(\Theta_w - \Theta_g) \quad (11)$$

with boundary conditions

$$X = 0, \quad \Theta_w = 0, \quad (12)$$

$$X = -1, \quad \Theta_w = 1, \quad (13)$$

$$X = -1, \quad \Theta_g = 1. \quad (14)$$

Equations (10) and (11) can be reduced to a normal system of three first-order linear homogeneous equations. Solving this system by the usual techniques, we obtain the following relations:

$$\Theta_w = C_1\exp(\gamma_1 X) + C_2\exp(\gamma_2 X) + C_3\exp(\gamma_3 X), \quad (15)$$

$$\Theta_g = C_1 a_1\exp(\gamma_1 X) + C_2 a_2\exp(\gamma_2 X) + C_3 a_3\exp(\gamma_3 X), \quad (16)$$

where

$$a_i = \frac{K^2}{K^2 + H\gamma_i}, \quad (17)$$

and the values of γ are determined from the characteristic equation

$$\gamma^3 + \frac{K^2}{H}\gamma^2 + (K^2 + \beta^2)\gamma - \frac{K^2\beta^2}{H} = 0. \quad (18)$$

At $\beta^2/H < 0.4$ approximate values of the roots of Eq. (18) can be determined from the formulas

$$\gamma_1 = 0.5H - \sqrt{(0.5H)^2 + \beta^2}, \qquad (19)$$

$$\gamma_{2,3} = -(K^2/2H) \pm \sqrt{(K^2/2H)^2 + (K^2 + \beta^2)}. \qquad (20)$$

The values of the constants in (15) and (16) are found from boundary conditions (12)–(14).

In most cases of practical importance

$$\exp(\gamma_3) \ll 1, \qquad (21)$$

$$\frac{\gamma_2 \exp(-\gamma_2)}{\gamma_3} \ll 1. \qquad (22)$$

With these conditions Eq. (15) takes the form

$$\Theta_w = -\left(1 + \frac{\beta^2\gamma_2}{K^2\gamma_3}\right) \frac{\exp(\gamma_1 X) - \mathrm{epx}(\gamma_2 X)}{\exp(-\gamma_1) - \exp(-\gamma_2)} + \frac{\beta^2\gamma_2}{K^2\gamma_3} \exp[(1 + X)\gamma_3]. \qquad (23)$$

In the section x = −L the dimensionless heat flow along the neck

$$\frac{Q_L}{Q_T} = \left(1 + \frac{\beta^2\gamma_2}{K^2\gamma_3}\right) \frac{-\gamma_1 + \gamma_2 \exp(\gamma_1 - \gamma_2)}{1 + \exp(\gamma_1 - \gamma_2)} + \gamma_2 \frac{\beta^2}{K^2}, \qquad (24)$$

where

$$Q_T = \frac{\lambda_w f}{L}(T_0 - T_L).$$

Given $\beta^2 < 0.1$ and assumptions (21) and (22), we can neglect the heat transfer at the outer surface of the neck. In this case

$$\frac{Q_L}{Q_T} = \frac{\gamma_2}{\exp(\gamma_2) - 1}. \qquad (25)$$

In [1] the following relation is given for the case of perfect heat transfer:

$$\frac{Q_L}{Q_T} = \frac{H}{\exp(H) - 1}. \qquad (26)$$

The external forms of Eqs. (25) and (26) are perfectly analogous, but the factors associated with convective heat transfer are taken into account by introducing the quantity γ_2 instead of the parameter H.

Solving Eqs. (1) and (2) for the case of perfect heat transfer, we obtain the following relation for determining the heat flow along the neck:

$$\frac{Q_L}{Q_T} = A\,\mathrm{cth}(A) - 0.5H, \qquad (27)$$

where

$$A = \sqrt{(0{,}5H)^2 + \beta^2}.$$

When $\beta = 0$, Eq. (27) reduces to Eq. (26).

Using (12), (15), and (16), we can determine the temperature difference between the gas and the walls at the "hot" end of the neck at X = 0. With assumptions (21) and (22) we have

$$\vartheta_0 = \frac{T_0 - T_{g0}}{T_0 - T_L} = \frac{(1 + \gamma_2\beta^2/\gamma_3 K^2)\exp(\gamma_1)}{\left[1 + \frac{K^2}{H\gamma_2}\right][1 - \exp(\gamma_1 - \gamma_2)]}. \qquad (28)$$

On the basis of the relations obtained we arrive at the following conclusions:

a) at $\beta > 1$ heat transfer at the outer surface of the neck leads to a considerable increase in axial heat flow;

b) in the case of effective heat transfer between the gas and the walls, as the gas flow rate increases the heat flow along the neck tends to a minimum value, at which it ceases to depend on the gas flow rate.

This minimum value of the heat flux is approximately equal to

$$\lim_{H \to \infty}\left(\frac{Q_L}{Q_T}\right) = K\left[\frac{\beta^2}{K^2} + \exp(-K)\right]. \qquad (29)$$

To check these relations we carried out experiments on a special apparatus (Fig. 1).

The apparatus was so designed that it was possible to determine directly the heat flow along neck 6 from the vaporizability of the cryogenic liquid in control chamber 1. Protective chamber 2, filled with the same cryogenic liquid as the control chamber, was provided to take care of secondary heat fluxes.

The experimental data for liquid nitrogen and hydrogen are presented in Fig. 2 for a neck made of Cr18Ni10Ti steel with dimensions D × δ = 18 × 0.3 mm. As the neck insulation we used aluminized mylar film with intermediate layers of glass cloth.

In the case of radiative heat transfer at the surface of the neck the expression for β^2, obtained from the

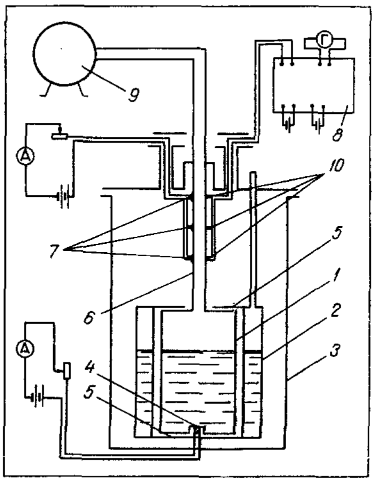

Fig. 1. Diagram of the experimental apparatus: 1) control chamber; 2) protective chamber; 3) outer shell; 4) inner heater; 5) protective disks; 6) neck; 7) neck heaters; 8) potentiometer circuit; 9) gas counter; 10) thermocouples.

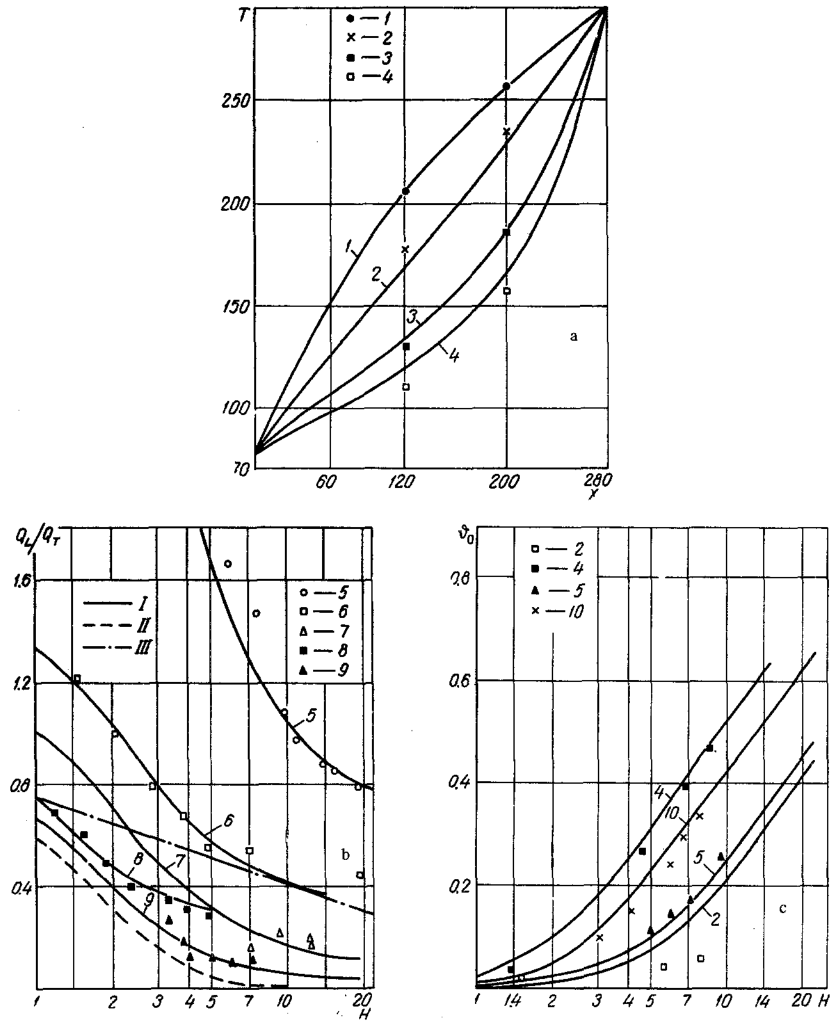

Fig. 2. Comparison of calculated and experimental data (x in mm, T in °K): a) temperature distribution in nitrogen experiments; b) dimensionless heat flow along neck; c) dimensionless temperature difference at "hot" end of neck: I) curves calculated from Eqs. (23), (27), (32), and (33), respectively; II) from Eq. (26); III) according to the data of [4]; 1) m = 0; 2) $1.5 \cdot 10^{-3}$ g/sec; 3) $3.53 \cdot 10^{-3}$; 4) $5.17 \cdot 10^{-3}$; 5) N_2, L = 0.28 m; 6) N_2, L = 0.28; 7) H_2, L = 0.28; 8) N_2, L = 0.12; 9) H_2, L = 0.12; 10) N_2, L = 0.12; 6–9) vacuum multilayer neck insulation; 1 and 10) high-vacuum neck insulation.

heat balance, has the form

$$\beta^2 = \frac{\sigma\varepsilon_w \pi D_w (T_0^4 - T_{av}^4)}{(T_0 - T_{av})\lambda_w f} L^2. \qquad (30)$$

As the quantity T_{av} it is possible to take the mean integral temperature of the neck walls, which is approximately equal to

$$T_{av} = T_0 + (T_0 - T_L) \times \left[\frac{1 - \exp(\gamma_1)}{\gamma_1} + \frac{\exp(\gamma_1) - \exp(\gamma_1 - \gamma_2)}{\gamma_2}\right]. \qquad (31)$$

Solving Eqs. (29) and (30) jointly by successive approximation, we find the value of β^2.

In analyzing the experimental data the values of the physical constants of the walls and the gas were taken at the mean wall temperature. The axial heat flow through the gas was taken into account by introducing instead of $\lambda_w f$ the quantity $\lambda_w f + \lambda_g F$. Since in all the experiments the gas flow was laminar, the value of Nu was taken equal to 3.66.

At small gas flow rates the experimental data are in good agreement with the theoretical values both with respect to temperature distribution and with respect to the values of the axial heat flows.

At large gas flow rates the theoretical values of the heat flows along the neck exceed the experimental values by 15–40% depending on the value of β and K.

These discrepancies are due to the assumptions made in formulating and solving the problem.

An increase in the value of the Nusselt number has only a very slight effect on the temperature distribution along the neck, but leads to an underestimation of the axial heat flows.

In the experiments the parameters were varied within the following limits:

$$0.3 \leq \beta^2 \leq 10, \quad 1 < H < 20,$$
$$13.5 < K^2 < 480; \quad T_{av} > 100° \text{K}.$$

Under these conditions good agreement between the experimental and calculated values of the heat fluxes can be obtained if it is assumed that

$$\frac{Q_L}{Q_T} = 0.9\left[-\gamma_1 + \gamma_2 \exp(\gamma_1 - \gamma_2) + \gamma_2 \frac{\beta^2}{K^2}\right]. \quad (32)$$

At $\beta^2 < 0.1$ it is possible to use Eq. (25) with conditions (21) and (22). The experimental values of the temperature difference between the walls and the gas at the hot end of the neck are well expressed by the relation

$$\vartheta_0 = \frac{\exp(\gamma_1)}{(1 + 2K^2/H\,\gamma_2)\left[1 - \exp(\gamma_1 - \gamma_2)\right]}. \quad (33)$$

The calculated and experimental data are compared in Fig. 2.

NOTATION

T is the temperature; Q is the heat flux; m is the gas flow rate; λ is the thermal conductivity; c is the isobaric specific heat of the gas; f is the cross-sectional area of the neck walls; F is the clear cross section; D is the diameter; L is the length of the neck; α is the heat transfer coefficient; ε is the emissivity; σ is the Stefan-Boltzmann constant. Subscripts: w—wall; g—gas; 0—ambient medium or section x = 0; L—cryogenic liquid or section x = L; in—insulation.

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